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Systematics in the structure of low-lying, non-yrast band-head configurations of strongly deformed nuclei

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Abstract. A systematic application of the pseudo-$SU(3)$ model for a sequence of rare earth nuclei demonstrates that an overarching symmetry can be used to predict the onset of deformation as manifested through low-lying collective bands. The results also show that it is possible to obtain a unified description of members of the yrast band and the $K^\pi = 2_1^+$ and $K^\pi = 0_2^+$ excited bands by using a classification scheme based on particle occupation numbers in the valence shells. The scheme utilizes an overarching $Sp(4,R)$ symplectic framework. The nuclei that are considered belong to the $F_0 = 0$ and $F_0 = 1$ symplectic multiplets of the (50,82–82,126) shell.

PACS. 21.60.Fw Models based on group theory – 21.10.Re Collective levels – 27.70.+q $150 \leq A \leq 189$

1 Introduction

The behavior of first excited $K^\pi = 0_2^+$, and $K^\pi = 2^+$ bands in deformed even-even nuclei is investigated empirically in the rare-earth region, where the nuclei are ordered in F-spin multiplets of a $Sp(4,R)$ classification scheme [1]. The energy levels of the ground state (g.s.) $J = 2_1^+$, first excited $J^\pi = 0^+_{K^\pi=0_2^+}$, $J^\pi = 2^+_{K^\pi=2_1^+}$ states of nuclei that belong to the $F_0 = 0 = 1/2(N^\pi - N^\nu)$ multiplet (where N^π and N^ν are the number of valence proton and neutron pairs) are plotted in fig. 1. The energies of the same set of levels for nuclei belonging to the $F_0 = 1$ multiplet are plotted in fig. 2. Nuclei with $F_0 = 0$ have equal numbers of valence proton and neutron pairs. The others with $F_0 = 1$ vary by two pairs of protons. We want to reproduce and interpret microscopically this complex and varying behavior by an application of the algebraic shell model with pseudo-$SU(3)$ symmetry.

2 Model space and interactions

The building blocks of the model are the pseudo-$SU(3)$ proton and neutron states having pseudo spin zero, which describe the even-even nucleus. The many-particle states are built as pseudo-$SU(3)$ coupled states with a well-defined particle number and total angular momentum [2,3].

The Hamiltonian that is appropriate for the description of nuclei being considered includes spherical single-particle terms for both protons and neutrons, H^σ_{sp}; proton and neutron pairing terms, H^σ_P; an isoscalar quadrupole-quadrupole interaction, $Q\cdot Q$; and four smaller "rotor-like" terms that preserve the pseudo-$SU(3)$ symmetry:

$$H = H^\pi_{sp} + H^\nu_{sp} - G_\pi\ H^\pi_P - G_\nu\ H^\nu_P - \frac{1}{2}\ \chi\ Q\cdot Q + a\ J^2 + b\ K_J^2 + a_3\ C_3 + a_s\ C_2\,, \quad (1)$$

where C_2 and C_3 are the second and third order invariants of $SU(3)$, which are related to the axial and triaxial deformation of the nucleus. The single-particle energies are calculated in the standard form with standard values for coefficients $D_{\pi[\nu]}$ and $C_{\pi[\nu]}$ from [4]:

$$H^\sigma_{sp} = \sum_{i_\sigma} \left(C_\sigma l_{i_\sigma}\cdot s_{i_\sigma} + D_\sigma l^2_{i_\sigma}\right), \quad (2)$$

where σ stands for protons (π) or neutrons (ν).

The calculations assumed fixed values [5] for pairing ($G_\pi = 21/A$, $G_\nu = 17/A$), as well as for the quadrupole-quadrupole interaction strength ($\chi = 35A^{-5/3}$). The other interaction strengths were varied to give a best fit to the band heads of the first excited $K^\pi = 0^+$, $K^\pi = 2^+$ and $K^\pi = 1^+$ bands, as well as the moment of inertia of the g.s. band [6]. Explicitly, the term proportional to K_J^2 breaks the $SU(3)$ degeneracy of the different K bands, the J^2 term represents a small correction to fine tune the moment

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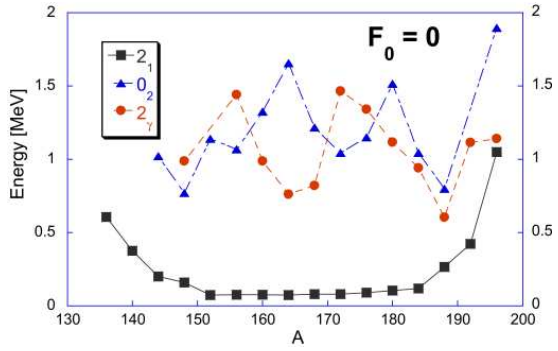

Fig. 1. The experimental energy values [7] of the $J^\pi = 2^+_1$, $J^\pi = 0^+_{K^\pi=0^+_2}$, and $J^\pi = 2^+_{K=2^+_1}$ states in a sequence of nuclei for which the numbers of valence protons and neutrons are the same ($F_0 = 0$).

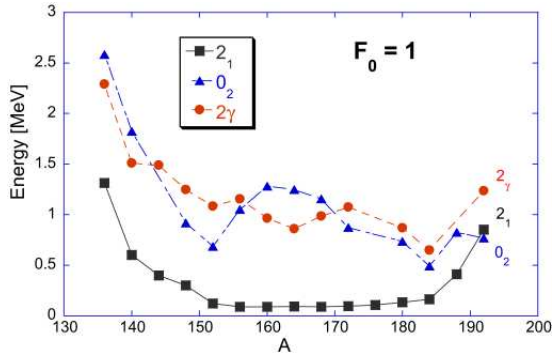

Fig. 2. The experimental energy values [7] of the $J^\pi = 2^+_1$, $J^\pi = 0^+_{K^\pi=0^+_2}$, and $J^\pi = 2^+_{K=2^+_1}$ states in a sequence of nuclei for which the difference in the numbers of valence proton and neutron pairs is two ($F_0 = 1$).

of inertia, and the last term, C_2, is introduced to distinguish between $SU(3)$ irreps with λ and μ both even from the others with one or both odd, hence fine tuning the energy of the first excited $K^\pi = 1^+$ state. Within this framework, the splitting and mixing of the pseudo-$SU(3)$ irreps are generated by the proton and neutron single-particle terms ($H^{\pi/\nu}_{sp}$) and the pairing interactions. This mixing plays an important role in the reproduction of the behavior of the low-lying collective states in deformed nuclei [6].

3 Results and conclusions

The experimental and calculated energies of the $J^\pi = 2^+_{\rm g.s.}$, $J^\pi = 0^+_{K^\pi=0^+_2}$, and $J^\pi = 2^+_{K^\pi=2^+_1}$ states in the deformed nuclei from the ones shown in figs. 1 and 2 are compared in tables 1 and 2. We calculated also all the states with $J \leq 8$ within these three bands.

The calculated results are in very good agreement with experiment [6,7]. The main reason for obtaining the position of each collective band with respect to each other, as well as of each level within the band is the specific content of the obtained $SU(3)$ irreps into the collective states, which is related to their deformations. For nuclei from table 1, in the middle of the shell, the ground and the γ band belong to the same (λ, μ) with $\lambda > \mu$. At the limits of the deformed region the ground band states have oblate deformation ($\lambda < \mu$) and the $K^\pi = 0^+_2$ and the $K^\pi = 2^+_1$ bands are mixed in the same $SU(3)$ irrep [6]. The analysis for nuclei from table 2 is under investigation.

The correct description of collective properties of first excited $K^\pi = 0^+_2$, and $K^\pi = 2^+$ states is a result of representation mixing and state deformation induced by the model Hamiltonian. This study shows that pseudo-spin zero neutron and proton configurations with relatively few pseudo-$SU(3)$ irreps with largest $C2$ values suffices to yield good agreement with known experimental energies.

Table 1. Energy values for the $J^\pi = 2^+_1$, first excited $J^\pi = 0^+_{K^\pi=0^+_2}$ and $J^\pi = 2^+_{K^\pi=2^+_1}$ states for seven nuclei in the $F_0 = 0$ multiplet. The experimental values [7] are given in parenthesis. The numbers of valence proton and neutron pairs are given in the second and third columns.

Nucleus	N^π	N^ν	$E(2_1)$ Th. (Exp.) [MeV]	$E(0^+_{K=0^+_2})$ Th. (Exp.) [MeV]	$E(2^+_{K^\pi=2^+_1})$ Th. (Exp.) [MeV]
^{152}Nd	5	5	0.082 (0.073)	1.14 (1.14)	1.31 (1.38)
^{156}Sm	6	6	0.078 (0.076)	1.07 (1.07)	1.45 (1.47)
^{160}Gd	7	7	0.085 (0.075)	1.33 (1.33)	0.99 (0.82)
^{164}Dy	8	8	0.073 (0.073)	1.67 (1.66)	0.76 (0.76)
^{168}Er	9	9	0.089 (0.080)	1.21 (1.22)	0.81 (0.82)
^{172}Yb	10	10	0.081 (0.078)	1.04 (1.04)	1.49 (1.47)
^{176}Hf	11	11	0.178 (0.088)	1.15 (1.15)	1.26

Table 2. Same as in table 1 for five nuclei with $F_0 = 1$.

Nucleus	N^π	N^ν	$E(2_1)$ Th. (Exp.) [MeV]	$E(0^+_{K=0^+_2})$ Th. (Exp.) [MeV]	$E(2^+_{K^\pi=2^+_1})$ Th. (Exp.) [MeV]
^{156}Gd	7	5	0.094 (0.089)	1.06 (1.05)	1.14 (1.15)
^{160}Dy	8	6	0.092 (0.087)	1.30 (1.28)	0.99 (0.97)
^{164}Er	9	7	0.091 (0.091)	1.25 (1.25)	0.94 (0.86)
^{168}Yb	10	8	0.088 (0.088)	1.16 (1.15)	0.98 (0.98)
^{172}Hf	11	9	0.119 (0.095)	0.87 (0.87)	1.08 (1.07)

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